Composites Science and Technology 62 (2002) 819–830
COMPOSITES SCIENCE AND TECHNOLOGY
www.elsevier.com/locate/compscitech

Crack bifurcation features in laminar specimens with fixed total thickness

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Received 20 September 2001; received in revised form 8 February 2002; accepted 12 February 2002

Abstract

A method to analyze the crack bifurcation behavior of symmetric two-component layered composites had been developed. With the method it is possible to compute the layer thickness at which cracks bifurcate. The thickness necessary for cracks to bifurcate depends on the elastic constants of the layers, the number of layers, the difference of thermal expansion coefficients and the temperature gradient. Further, the thickness ratios at which crack bifurcation can occur for layers with tensile and compressive residual stresses were computed. In the work the important case of specimens with fixed total thickness is considered. There are some features of crack bifurcation under these conditions.

Keywords: A. Ceramic-matrix composites (CMCs); A. Layered structures; B. Mechanical properties; B. Fracture; C. Residual stress

1. Introduction

The strategy of ceramic strengthening is usually associated with the design of ceramic composites to enable different mechanisms of fracture energy absorption or dissipation. In addition, other properties can be modified by a suitable microstructure design in composites, including those which cannot be combined in a single-phase material. In functional gradient materials even more complex microstructural architectures have to be developed. Different structural elements such as particles, whiskers, filaments, platelets or laminates are combined to tailor properties to different service requirements.

Composites are complicated systems characterized by a host of interdependent parameters. The theoretical prediction of the mechanical behavior provides information related to the failure of such materials. The strength of ceramic-matrix layered composites depends on the properties of the separate layers and also the interactions between different layers [1]. Ceramic materials show a lot of outstanding physical and chemical properties, which make them interesting for many engineering purposes. However, their more intensive technical application is restricted by their brittleness. A key feature that imparts good mechanical properties in multilayer systems is the ability to deflect cracks.

There are two different mechanisms of the crack deflection. Cracks that form in one layer can be deflected either along weak interfaces with adjacent layers or into layers with compressive residual stresses. Essential for the first mechanism is matrix/interface strength ratio. The presence of weak interfaces transverse to a crack growing in a brittle material causes the crack to be deflected with consequent increase in the resistance to crack growth [2]. Note that presence of absence of internal stresses is not necessary condition for operation of this mechanism. Crack bifurcation is the second mechanism of crack propagation operating in layer with compressive residual stresses. Key feature of the bifurcation phenomenon is its correlation with the extension of an edge crack at the surface of a layer containing compressive residual stresses. The presence of the

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0266-3538/02/$ - see front matter
PII: S0266-3538(02)00040-4

compressive stresses is necessary condition for bifurcation [3]. The formation of weak interfaces with controlled strength is a very difficult technological problem. Furthermore, weak interfaces usually also increase high temperature corrosion due to the high defect density. Therefore, crack bifurcation in layers with compressive residual stresses is the preferred mechanism to improve laminates.

In previous work Oechsner et al. [3] have observed crack bifurcation in laminar ceramic composites when crack entered thin Al_2O_3 layers sandwiched between thicker layers of $Zr(12Ce)O_2$.The Al_2O_3 layers contained a biaxial, residual compressive stress of 1.8 GPa developed due to differential contraction upon cooling from the processing temperature. The $Zr(12Ce)O_2$ layers were nearly free of residual tensile stresses because they were much thicker than the Al_2O_3 layers. Despite of the fact that the residual stresses were nearly identical in all Al_2O_3 layers, crack bifurcation was observed only when the layer thickness was greater than 70 μm.

Sanchez-Herencia et al. [4] have reported the bifurcation results in laminates containing a thin layer of pure zirconia (MZ) and Y-TZP (TZ) mixtures sandwiched between two thick layers. Residual compressive stresses in the thin layer was produced by the tetragonal to monoclinic phase transformation of the ZrO_2 in the thin layer. The strain produced by this transformation is much greater than that produced by differential thermal expansion. The transformation strain and the temperature where the transformation occurred were varied by mixing pure ZrO_2 powder (MZ) with $Zr(Y)O_2$ powder containing 3 mol% Y_2O_3 (TZ). It was found in [4] that crack bifurcation during flexural loading occurred for thin layer compositions containing $\geqslant 0.4$ volume fraction MZ powder, when the thickness of the thin layer was between 50 and 150 μm. Crack bifurcation was not observed in thinner layers. The capability to create residual stresses using the zirconia phase transformation was also probed using the alumina/50 vol.% MZ/alumina system [5]. In this system the critical thickness for crack bifurcation was of 25 μm.

The results obtained in [3,4] show that crack bifurcation phenomenon depends on the layer thickness in addition to residual stress. The goal of the present work is the development of an analysis method for two-component ceramic matrix layered composites with crack bifurcation in layers with compressive residual stresses controlled by the layer thickness. In the work the important case of specimens with fixed total thickness is considered. There are some features of crack bifurcation under these conditions. The method is necessary for an optimal design of processing parameters for fabrication of such composites. Experimental validation of these analytical results was made by using the experimental results reported earlier in the literature [4]. The paper is focused on mechanism of crack bifurcation controlled by difference of residual stresses within the body and those at the free surface of the layered material. The features of crack propagation mechanism controlled by crack interaction with interfaces are not considered.

2. Thermal residual stresses in laminates

In this work two-component layered composites with symmetric macrostructures are considered (Fig. 1). The layers alternate, with exposed faces being of the same component. Thus the total number of layers N in the composite sample is odd. In Fig. 1 the layers of the first component including the two external (outside) layers are designated with the index 1 ($j=1$), and those of the second component (internal) with index 2 ($j=2$). The number of layers designated with index 1 is $(N+1)/2$ and the number of layers designated with index 2 is $(N-1)/2$. All layers of each component have the same constant thickness.

In this work the mechanical behavior of a layered beam (sample) with a rectangular cross section with the height (or total thickness of specimen) h and the width b is considered. It is assumed that the two components have different thermal expansion coefficients. Therefore the difference in temperature ΔT between the actual temperature and the temperature at which the layers constituting the material were joined is the important initial parameter which determines the residual thermal stress in the layers.

Characteristics of the individual structural components such as Young's modulus, strength, fracture toughness, and thermal expansion coefficients affect the composite failure process. The effective elastic modulus of the layered composite (Fig. 1) is determined by the Young's modulus E_j ($j=1$, 2) and the thickness l_j ($j=1$, 2) of the layers. Further, the strength of j-th component of layered composite is σ_{cj}, the fracture toughness is K_{cj}, and the thermal expansion coefficient is α_{Tj}.

During cooling of the sample the deformation difference, due to the different thermal expansion coefficients, is accommodated by creep as long as the temperature is

Fig. 1. Two-component layered composite: (1) layers of the first component including two external (outside) layers; (2) layers of the second component (internal).

high enough. Below a certain temperature, called the "joining" temperature, the different components become bonded together and internal stresses appear. The "joining" temperature is usually the value that is known only approximately. It is necessary the additional experimental investigations of this parameter. In each layer, the total deformation after sintering is the sum of an elastic component and of a thermal component [6]. In the case of a perfectly rigid bonding between the layers, the total deformation will be the same for all the layers:

$$e_j = \frac{\sigma_{rj}}{E'_j} + a_{Tj}\Delta T = \text{const} \tag{1}$$

where σ_{rj} is the residual stress in the j-th component, $E'_j = E_j/(1-\nu_j)$, ν_j is Poisson's ratio of the j-th component.

The force balance requires (in normal stresses):

$$\sum_j \sigma_{rj} f_j = 0 \tag{2}$$

where f_j is the volume fraction of j-th component.

For two-component material, f_j is:

$$f_1 = \frac{(N+1)l_1}{2h} \text{ and } f_2 = \frac{(N-1)l_2}{2h}$$

The combination of Eqs. (1) and (2) results in:

$$\sigma_{r1} = \frac{E'_1 E'_2 f_2(\alpha_{T2}-\alpha_{T1})\Delta T}{E'_1 f_1 + E'_2 f_2} \tag{3}$$

and

$$\sigma_{r2} = \frac{E'_2 E'_1 f_1(\alpha_{T1}-\alpha_{T2})\Delta T}{E'_1 f_1 + E'_2 f_2} \tag{4}$$

In this paper we will consider components marked with index 1 as the ones with tensile residual stress and those with index 2 as the ones with compressive residual stress. It should be also noted that the expression (3) and the expression (4) can be generalized for non-thermal source of residual stress. To transform these equations it is only necessary to do the simple replacement of $(\alpha_{T1}-\alpha_{T2})\Delta T$ by the some strain mismatch $\Delta\varepsilon$ between layers of type 1 and type 2.

3. Determination of crack deflection regime

To understand the thinking that led to experiments concerning crack bifurcation, consider a crack propagating through a laminate as shown in Fig. 2. When a crack propagates through the laminate, it creates a free surface. When it approaches a layer that is under a residual compressive stress, σ_r, it creates a situation similar to the free edge of a laminate [3,7]. The tensile stress, σ_T, which now arises due to the introduction of the free surface, may cause the propagating crack to bifurcate into the compressive layer. Because this behavior is similar to the edge-cracking problem, a critical layer thickness is expected below which no crack bifurcation will occur.

Ho et al. [7] observed that the occurrence of edge cracks was dependent on the thickness of the compressive layer and the magnitude of the residual compressive stress in this layer. They developed a strain energy release rate function for a crack in this localized tensile stress field. The thickness for which the crack bifurcation occur can be determined from the expression:

$$l_2 \geqslant \frac{G_{c2}E_2}{0.34(1-\nu_2^2)\sigma_{r2}^2}, \tag{5}$$

where G_{c2} is the critical strain energy release rate of the second composite component. It should be noted that $G_{c2}E_2/(1-\nu_2^2) = K_{c2}^2$.

Eqn. 4 can be easily transformed into the following form using the expressions for f_1 and f_2:

$$\sigma_{r2} = \frac{E_2\Delta\alpha\Delta T}{(1-\nu_2)\left(1+\frac{E_2}{E_1}\left(\frac{N-1}{N+1}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)\frac{l_2}{l_1}\right)} \tag{6}$$

where $\Delta\alpha = \alpha_{T1} - \alpha_{T2}$.

Criterion (5) is transformed using (6) into:

$$\frac{l_2}{\left(1+\frac{E_2}{E_1}\left(\frac{N-1}{N+1}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)\frac{l_2}{l_1}\right)^2} \geqslant \frac{(1-\nu_2)^2 K_{c2}^2}{0.34(E_2\Delta\alpha\Delta T)^2} \tag{6a}$$

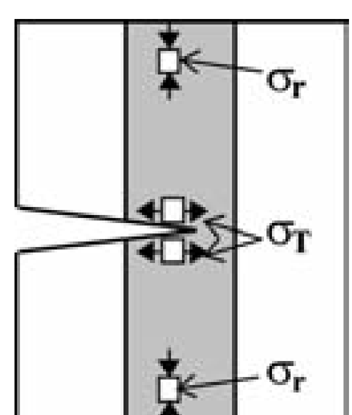

Fig. 2. Schematic of a propagating crack in a laminate under residual compressive stress.

and then

$$l_2 - l_c\left(1 + \frac{E_2}{E_1}\left(\frac{N-1}{N+1}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)\frac{l_2}{l_1}\right)^2 \geqslant 0,$$

where $l_c = \frac{(1-\nu_2)^2 K_{c2}^2}{0.34(E_2 \Delta\alpha \Delta T)^2}$ is the characteristic layer thickness. The dependence of characteristic size l_c on the thermal expansion factors difference $\Delta\alpha$ for various temperature differences ΔT is shown in Fig. 3. The characteristic size increases with decreasing ΔT and $\Delta\alpha$.

Using the substitution $a = \frac{E_2}{E_1}\left(\frac{N-1}{N+1}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)\frac{l_c}{l_1}$ we obtain:

$$\frac{a^2}{l_c} l_2^2 - (1-2a)l_2 + l_c \leqslant 0. \tag{7}$$

In the case of fixed number of layers a is constant for given l_1. Then we have square equation for l_2. From (7) the upper and lower limits, $l_{b2}{}'$ and $l_{b2}{}''$, of the crack deflection area can be determined [8]:

$$l_{b2}{}',{}'' = \frac{1 - 2a \mp \sqrt{1-4a}}{2a^2} l_c. \tag{8}$$

The thickness of the second component layer for which crack bifurcation occurs can be determined in this case from the expression:

$$l_{b2}{}'(l_1) \leqslant l_2 \leqslant l_{b2}{}''(l_1)$$

The layer thickness regime where crack bifurcation occurs is shown in Fig. 4(a). These thicknesses depend on the elastic constants of the layers, the number of layers, the difference of thermal expansion factors and the temperature difference. For a given thickness of layers with tensile residual stresses there exists a thickness interval of the layers with compressive residual stresses where the crack bifurcation occurs. The regime boundaries for $E_2/E_1 = 1$ and for $E_2/E_1 = 2$ are presented in Fig. 4(b).

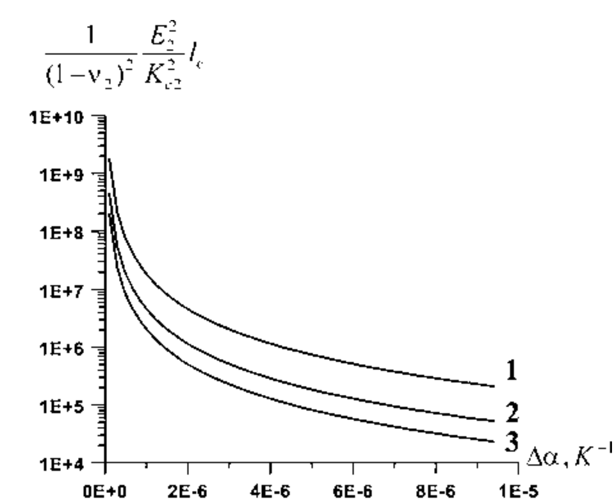

Fig. 3. Dependence of characteristic size l_c on the difference of thermal expansion factors $\Delta\alpha$ for various temperature differences: (1) $\Delta T = 400$ K; (2) $\Delta T = 800$ K; (3) $\Delta T = 1200$ K.

Eq. (8) results in the expression:

$$1 - 4a \geqslant 0 \text{ or } a \leqslant 0.25. \tag{9}$$

From (9) a criterion for the first component layer thickness can be obtained

$$l_1 \geqslant l_1^*, \tag{10}$$

where $l_1^* = 4\frac{E_2}{E_1}\left(\frac{N-1}{N+1}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)l_c$ is the characteristic thickness of the first component. No deflection occurs as long as the layer thickness of the first component are less than the characteristic thickness (Fig. 4a). The dependence of the first component's characteristic thickness l_1^* on the ratio E_2/E_1 is presented in Fig. 5.

In the case of fixed total thickness of specimen h the parameter a depends on l_1 and l_2. It should be noted that the total thickness of sample is

$$h = \frac{N+1}{2} l_1 + \frac{N-1}{2} l_2. \tag{11}$$

Then we obtain from (11) $h - l_1 = \frac{N-1}{2}(l_1 + l_2)$ and $h + l_2 = \frac{N+1}{2}(l_1 + l_2)$ that result in

$$\frac{N-1}{N+1} = \frac{h - l_1}{h + l_2}. \tag{12}$$

Then $a = \frac{E_2}{E_1}\left(\frac{h-l_1}{h+l_2}\right)\left(\frac{1-\nu_1}{1-\nu_2}\right)\frac{l_c}{l_1} = \frac{E_2}{E_1}\left(\frac{1-\nu_1}{1-\nu_2}\right)\left(\frac{\frac{h/l_c}{l_1/l_c} - 1}{(h/l_c)+(l_2/l_c)}\right)$ and using the substitutions

$$A = \frac{E_2}{E_1}\left(\frac{1-\nu_1}{1-\nu_2}\right)\left(\frac{h/l_c}{l_1/l_c} - 1\right),$$

$H = \frac{h}{l_c}$ and $l_2^* = \frac{l_2}{l_c}$ we have $a = \frac{A}{H + l_2^*}$.

Then from (7) we obtain

$$-A^2 \frac{l_2^{*2}}{(H + l_2^*)^2} + \left(1 - \frac{2A}{H + l_2^*}\right) l_2^* - 1 = 0$$

and after transformations

$$l_2^{*3} + (2H - (A+1)^2) l_2^{*2} + H(H - 2(A+1)) l_2^* - H^2 = 0. \tag{13}$$

It is cubic equation for l_2^* under given condition. Symmetric configuration considered can have minimal number of layers 3 and for fixed total thickness of specimen we have the following limitation for l_2^*:

$$0 < l_2^* \leqslant H - (2l_1/l_c).$$

The equation $l_2^* = H - (2l_1/l_c)$ determines upper boundary of bifurcation area in this case. The cubic eqn (13) can have one real solution, two real solutions or three real solutions which determine low boundary of bifurcation area. In our case there is the physical sense only for positive real solutions because l_2^* is layer thickness. In practice in most cases we have only two variants:

- for small l_1 there is no positive or real solution for l_2^*, no bifurcation is in sample;
- for l_1 more than certain limit there is one positive real solution for l_2^*, there is bifurcation if l_2 more than its solution under given conditions.

In the case of thermal source of residual stress the areas of crack deflection was calculated by using of the following values of thermoelastic parameters: (1) the difference of the actual temperature and "joining" one is $\Delta T = -1200$ K; (2) the elastic moduli of first and second component are $E_1 = 327$ GPa and $E_2 = 310$ GPa; (3) the Poisson's ratios of first and second component are $\nu_1 = 0.26$ and $\nu_2 = 0.27$; (4) the coefficients of thermal expansion of first and second component are $\alpha_1 = 3.55 \cdot 10^{-6}$ K^{-1} and $\alpha_2 = 3 \cdot 10^{-6}$ K^{-1} respectively. The fracture toughness of second component is $K_{c2} = 5.5$ MPa·m$^{1/2}$. The values of parameters was selected so as to correspond to layered system $Si_3N_4 + 20\%TiN$ (component 1)/Si_3N_4 (component 2) [8]. For given system the calculated value of characteristic layer thickness is $l_c = 1.267 \cdot 10^{-3}$ m.

The areas of crack deflection for the different fixed total thickness of specimen h are shown in Fig. 6. Solid lines are upper boundaries of the areas. Dashed curves are low boundaries of the areas. In such a way each deflection (bifurcation) area is situated between respective boundaries. The following thickness of specimen were considered for given laminate: (1) $h = 4$ mm; (2) $h = 12$ mm; (3) $h = 20$ mm; (4) $h = 28$ mm; (5) $h = 40$ mm.

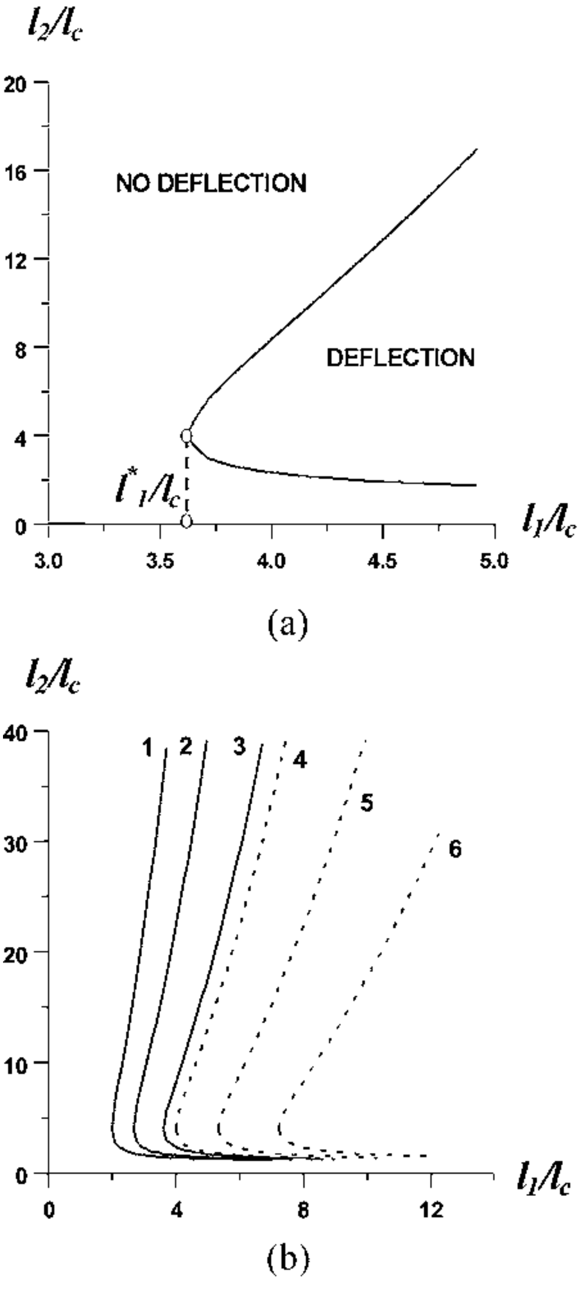

Fig. 4. Area of crack deflection (a) and boundaries (b) for $E_2/E_1 = 1$ and $N = 3$, $N = 5$, $N = 21$ (solid curves 1, 2, and 3), and for $E_2/E_1 = 2$ and $N = 3$, $N = 5$, $N = 21$ (dashed curves 4, 5, and 6) (the case of fixed number of layers).

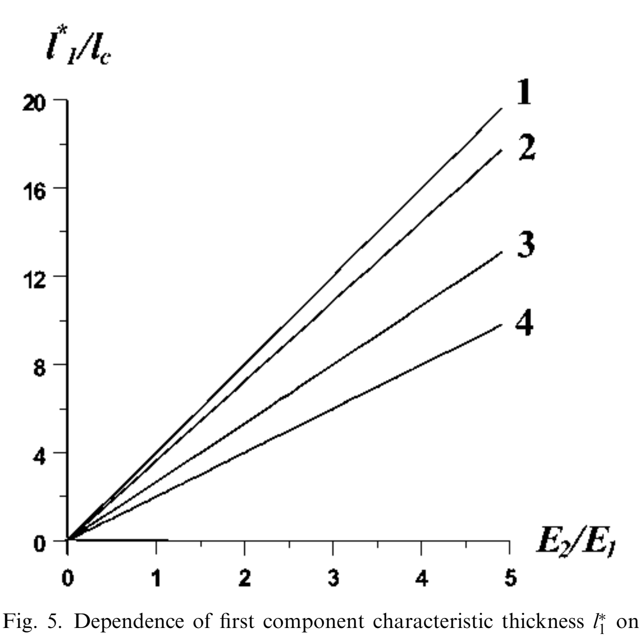

Fig. 5. Dependence of first component characteristic thickness l_1^* on ratio E_2/E_1 and $N = \infty$, $N = 21$, $N = 5$, $N = 3$ (curve 1, 2, 3, and 4 respectively).

It should be noted there is no bifurcation area for $h=4$ mm. In this case the low boundary of layer thickness to bifurcate is higher than the upper limitation due to fixed total thickness of sample. More thickness of specimen determines more deflection area.

For layered sample with fixed thickness the useful parameter can be calculated. It is the critical number of layers. It is the maximum number of layers in specimen to occur the bifurcation under given conditions. If the sample has more number of layers it is no bifurcation. Critical number of layers for the different fixed total thickness of specimen h is presented in Fig. 7. The above-mentioned thickness of sample are considered. It is well seen there are no real samples with bifurcation for $h=4$ mm. The maximum number of layers can be 9 for specimen with $h=40$ mm to occur the bifurcation. More thickness of specimen determines more number of laminar macrosructure variants to occur the bifurcation.

4. Comparison with experimental results

Laminate composites containing a thin layer (5–200 μm thick) sandwiched between two thick layers (2 mm thick) were fabricated in [4] by sequential slip casting to study crack bifurcation. The thin layer was formed with a mixture of a pure ZrO_2 powder (MZ) and a $Zr(Y)O_2$ powder containing 3 mol% Y_2O_3 (TZ). The sequential slip casting allowed the fabrication of layered materials where the thickness of each layers is controlled by the slurry casting period. The TZ powder, mixed with 0.05 volume fraction of Al_2O_3 powder, was used to form the thicker layers (≈0.25 cm) that sandwiched the thin layer in each specimen. The Al_2O_3 was used to add contrast and help distinguish the different layers in the scanning electron microscope. Different mixtures of MZ (0.35, 0.40, 0.45, 0.50, 0.60, to 1.0 volume fraction) and TZ (without added Al_2O_3) were used to form the thin layers in the different laminates.

Sequential slip casting was used to obtain layered plates (approximately 0.5 cm×7 cm×7 cm prior to densification) containing one thin layer sandwiched between two thicker layers, as well as monolitic specimens with the same composition as the thin layer in the laminates. To investigate the effect of layer thickness on the bifurcation phenomenon, different specimens were prepared to contain a single thin layer with a thickness between ≈5–150 μm. Monolitic bars (0.2 cm×0.5 cm×1 cm) were also formed with slurries containing specific MZ + TZ mixtures used to form laminated materials. These materials were used for phase identification and thermal expansion measurements to determine the strain and temperature associated with the tetragonal-to-monoclinic phase transformation during cooling. More detailed description of experimental procedure to study crack bifurcation can be found in Sanchez-Herencia et al. [4].

In the monolitic materials transformation temperature and the linear expansion associated with the transformation was found in [4] to decrease with increasing fraction of TZ. Laminates fabricated with a thin layer containing ⩾45% of MZ was shown to delaminate during cooling from their densification temperature

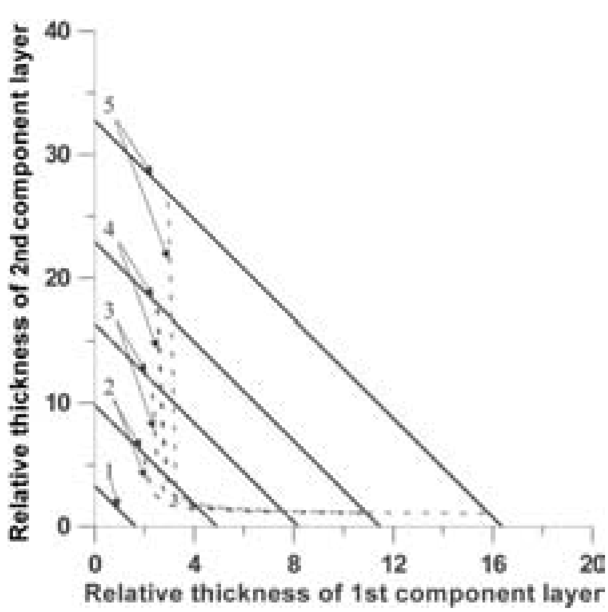

Fig. 6. Areas of crack deflection for the different fixed total thickness of specimen h. Solid lines are upper boundaries of the areas, dashed curves are low boundaries of areas. For the laminate considered, thermal residual stresses (see text) (1) $h=4$ mm; (2) $h=12$ mm; (3) $h=20$ mm; (4) $h=28$ mm; (5) $h=40$ mm.

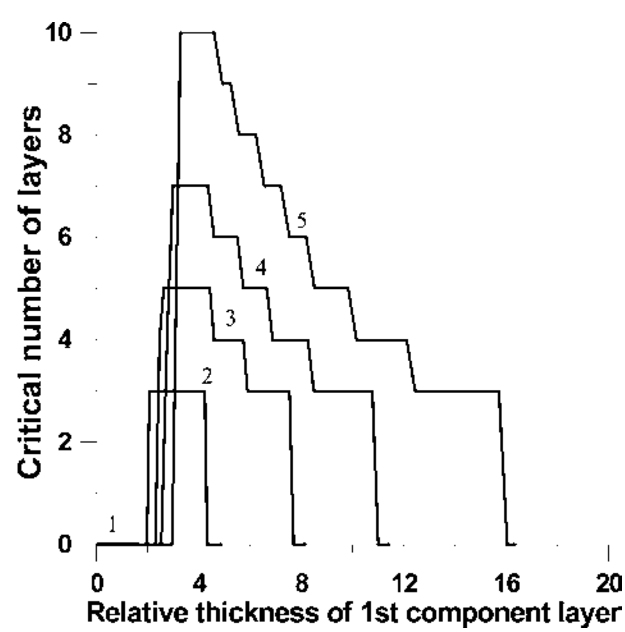

Fig. 7. Critical number of layers for the different fixed total thickness of specimen h. For the laminate considered, thermal residual stresses (see text) (1) $h=4$ mm; (2) $h=12$ mm; (3) $h=20$ mm; (4) $h=28$ mm; (5) $h=40$ mm.

when the layer was ⩾200 μm thick. Since the delamination crack propagated within the thin layer, it was assumed to be a special case of an edge crack that propagated through the specimen thickness. For thin layers with compositions containing >0.40 MZ, edge cracks were observed near the center line when their thickness was greater than a critical value. For specimens containing pure MZ, the edge crack was observed when the thin layer was ⩾60 μm. Although no single edge crack was observed for thinner layers, the surface of these thin layers was full of microcracks that were parallel to one another and to interface. No microcracks were observed when the thickness of this layer was 10 μm.

Fig. 8 shows SEM micrographs for crack propagation in specimen fabricated with thin layer containing 50 vol.% of m-ZrO_2. (a) where the crack propagates in thin layer without any bifurcation, and (b) where the bifurcated crack exhibits extensive propagation with the thin layer, then reinitiates failure.

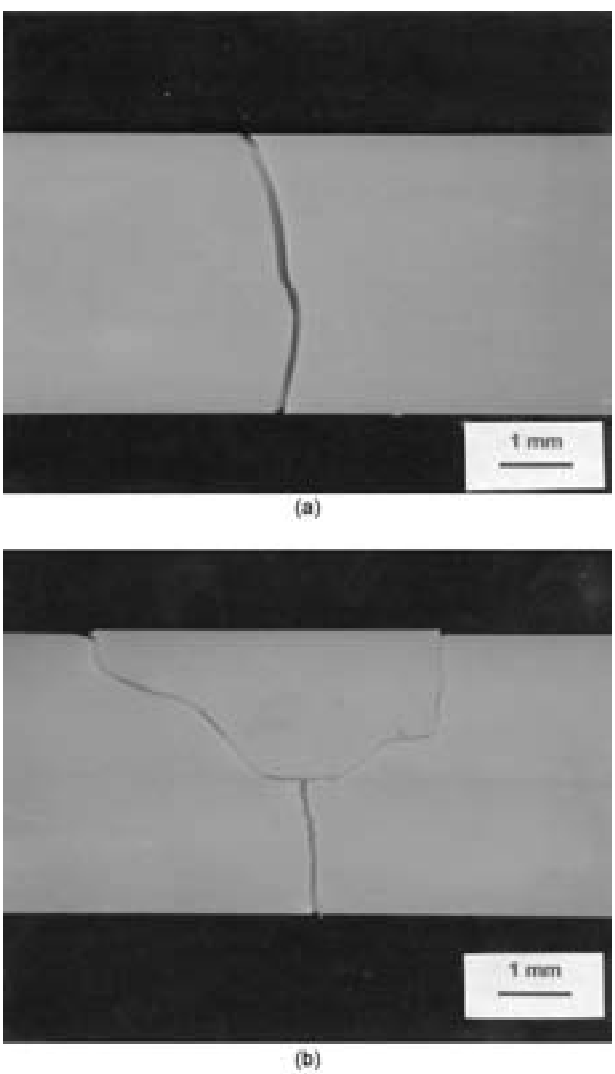

Fig. 8. SEM micrographs for character of crack propagation in specimen fabricated with thin layer containing 50 vol.% of m-ZrO_2: (a) thickness of thin layer is 50 μm (no bifurcation); (b) thickness of thin layer is 100 μm (extensive bifurcation).

Fig. 9 shows a plot of composition vs thin layer thickness where delamination, edge cracking and crack bifurcation was observed. One can see that edge cracking, which occurred during cooling, and crack bifurcation, which occurred during flexural loading, occurred for thin layer compositions containing ⩾40 volume fraction MZ powder, when the thickness of the thin layer was between ≈50 and 150 μm. Crack bifurcation was not observed in thinner layers.

The characteristic layer thicknesses l_c for laminates of the system ZrO_2/ZrO_2 along with other characteristic parameters calculated by using of expressions (7), (8), are listed in a table. Note that due to the fact that internal stresses in this system are generated by tetragonal-to-monoclinic phase transformation, expressions to calculate characteristic parameters of bifurcation have to be modified. Specifically, the expressions are derived from Eqs. (6a), (8), (13) simply by substituting $\Delta\varepsilon_0$ instead of $\Delta\alpha\Delta T$ ($\Delta\varepsilon_0$ is the linear transformation strain, i.e. different in length (per unit length) between inner and outer layers). It is the above-mentioned case of non-thermal source of residual stress. Here the linear transformation strain is the strain mismatch between layers. The values of the strain employed for calculation are taken from [4], Table 1. The values of other parameters are taken as [3,7]: $\nu_1 = \nu_2 = 0.25$, $E_1 = E_2 = 102$ GPa, $K_{c2} = 5$ MPa·m$^{1/2}$. It should be noted that ZrO_2 demonstrates the elastic modulus within interval 80–205 GPa depending on damage intensity. The

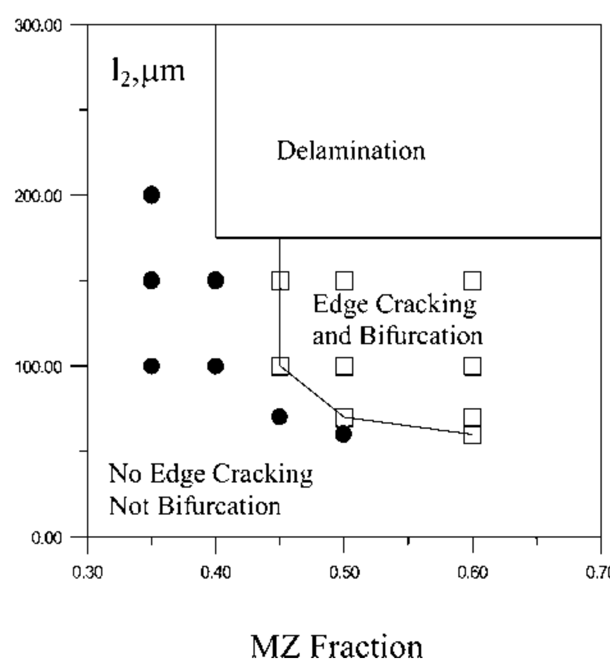

Fig. 9. Plot of thickness vs composition of the thin layer indicating the conditions observed for delamination, edge cracking and bifurcation [4].

observation of polished specimens containing thin layers show that the surface of the thin layer contains many microcracks which are presumed to form during the transformation [4]. These observations suggest that that the number density of microcracks, which would significantly decrease the elastic modulus and the residual, compressive stresses. Therefore the elastic modulus for calculation was chosen near the low boundary of known interval. The calculated boundaries of bifurcation areas and the points corresponding to specimens considered in [4] are presented in Fig. 10. The circles correspond to real laminar structures with different fixed total thickness of specimen h from [4]. Open circles correspond to the absence of bifurcation in experimental observations. Filled circles correspond to the bifurcation in experimental observations. Dashed lines are the low calculated boundaries of bifurcation areas. Solid lines are the upper calculated boundaries of bifurcation areas for respective fixed total thickness of specimen h. There is good correlation of experimental bifurcation and point location in calculated bifurcation area. It should be noted that only three layer specimens were considered in [4]. In such a way the points corresponding to real structure are situated on the upper boundaries of

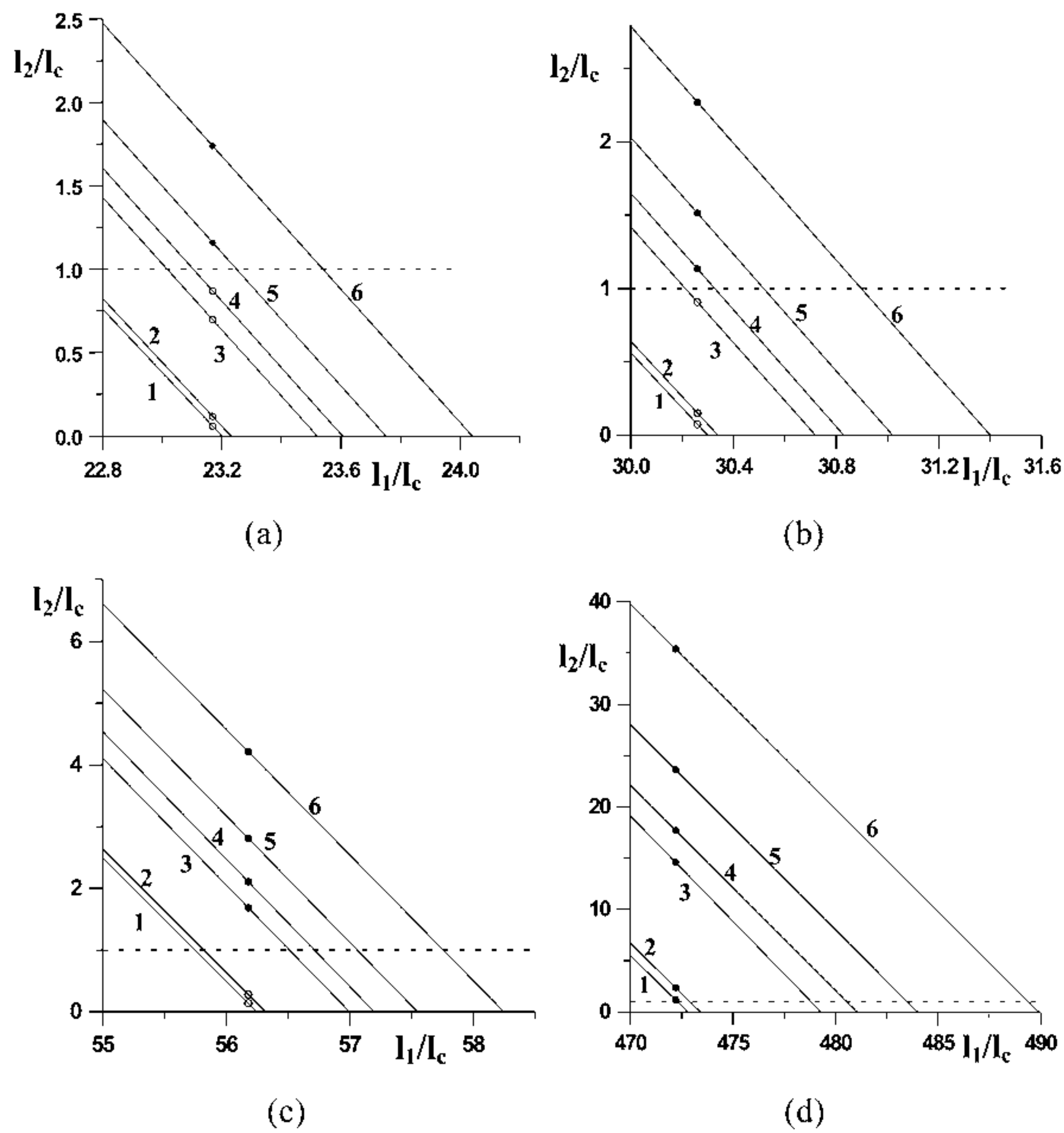

Fig. 10. Bifurcation areas for the different fixed total thickness of specimen h for the laminate considered in [4]. The circles correspond to real laminar structures from [4]. Open circles correspond to the absence of bifurcation in experimental observations. Filled circles correspond to the bifurcation in experimental observations. Dashed lines are the low calculated boundaries of bifurcation areas. Solid lines are the upper calculated boundaries of bifurcation areas. (a) $l_c = 86.3$ μm; (b) $l_c = 66.1$ μm; (c) $l_c = 35.6$ μm; (d) $l_c = 4.23$ μm. (1) $h = 4.005$ mm; (2) $h = 4.01$ mm; (3) $h = 4.06$ mm; (4) $h = 4.075$ mm; (5) $h = 4.1$ mm; (6) $h = 4.15$ mm.

Table 1
Characteristic parameters for crack bifurcation in ZrO_2/ZrO_2 layered composites (according to [4])

Internal layer thickness l_2 (μm)	MZ volume fraction in internal layer											
	0.45			0.50			0.60			1		
5	86.3	46.40	23.17	66.1	60.60	30.26	35.6	112.5	56.18	4.23	945.6	472.2
	4005	0.021	0.057	4005	0.016	0.075	4005	0.008	0.140	4005	0.001	1.180
	1.044	2171	–	1.033	3780	–	1.016	15 373	–	1.002	$9.9{\cdot}10^5$	+
10	86.3	46.46	23.17	66.1	60.68	30.26	35.6	112.6	56.18	4.23	946.8	472.2
	4010	0.021	0.115	4010	0.016	0.151	4010	0.008	0.280	4010	0.001	2.361
	1.044	2171	–	1.033	3780	–	1.016	15 373	–	1.002	$9.9{\cdot}10^5$	+
60	86.3	47.04	23.17	66.1	61.44	30.26	35.6	114.0	56.18	4.23	958.6	472.2
	4060	0.021	0.695	4060	0.016	0.907	4060	0.008	1.685	4060	0.001	14.16
	1.044	2171	–	1.033	3780	–	1.016	15 373	+	1.002	$9.9{\cdot}10^5$	+
75	86.3	47.21	23.17	66.1	61.66	30.26	35.6	114.4	56.18	4.23	962.1	472.2
	4075	0.021	0.868	4075	0.016	1.134	4075	0.008	2.106	4075	0.001	17.70
	1.044	2171	–	1.033	3780	+	1.016	15 373	+	1.002	$9.9{\cdot}10^5$	+
100	86.3	47.50	23.17	66.1	62.04	30.26	35.6	115.1	56.18	4.23	968.0	472.2
	4100	0.021	1.158	4100	0.016	1.513	4100	0.008	2.809	4100	0.001	23.61
	1.044	2171	+	1.033	3780	+	1.016	15 373	+	1.002	$9.9{\cdot}10^5$	+
150	86.3	48.08	23.17	66.1	62.80	30.26	35.6	116.5	56.18	4.23	979.8	472.2
	4150	0.021	1.737	4150	0.016	2.269	4150	0.008	4.213	4150	0.001	35.41
	1.044	2171	+	1.033	3780	+	1.016	15 373	+	1.002	$9.9{\cdot}10^5$	+

Table specification:

l_c, μm,	h/l_c,	l_1/l_c
h, μm,	a,	l_2/l_c
l_{b2}'/l_c,	l_{b2}''/l_c,	$+/-$[a]

[a] +, denotes there is bifurcation; –, denotes there is no bifurcation

bifurcation areas. The bifurcation takes place only when the respective point is above the low calculated boundaries of bifurcation area.

5. Discussion

We have obtained analytical expressions for the assessment of low and upper boundaries of layer thickness range where effect of bifurcation occurs in layers with compressive residual stresses. Accounting for similarity of the bifurcation behavior with edge-cracking problem, we have shown that there exists a parameter denoted by critical layer thickness below which no crack bifurcation occur. The parameter depends strongly on the processing temperature difference, elastic constants of the layers, the number of layers and the thermal expansion factors difference. The good correlation between the analytical predictions and experimental results on ZrO_2/ZrO_2 laminates provides confidence that the model, though rather simple, captures the essential features of the bifurcation phenomenon in brittle two-component symmetrical multilayer system. The model predictions are checked particularly for the multilayered system in which internal stresses are generated by phase transformation. Generalization of analitical expressions for prediction of the phenomenon in layered system with thermal source of internal stress is very simple.

Note that key assumption made in above analysis is that bifurcation is predominantly driven by internal stresses, due to phase transformation or thermal mismatch. One might be surprised if residual stresses did not play some role, but the question really is how great a role do they play?

For example, there are observations in the literature, e.g. in Clegg's laminates with porous interlayers [9], where the difference in expansion coefficients is zero although crack deflection occurs with the same distinctive features as that in Sanchez-Herencia et al. [4]. Investigated in [9] were ceramic laminates made from alternating layers of silicon carbide and silicon carbide containing a fugitive polymer, which could be pyrolysed to produce porous interlayers. So, both lamina and interlayer were made from the same material, resulting in avoiding any internal stresses due to differences in thermal expansion coefficients between the lamina and interlayer materials. A simple model was developed in [9] to describe the fracture behavior of porous solids and predict the volume fraction of porosity required to give crack deflection in the laminate. Taken into account in the model was the fact that defects in an interface ahead of a growing crack significantly affect the interfacial properties required for crack deflection. The key assumption of the model is that crack deflection occurs when the driving force for the growth of the interfacial crack equals the fracture energy of the interface at a lower load than that at which the driving force of the penetrating crack is equal to the fracture energy of the matrix. The crack-pore interaction is regarded in the model as providing an extra driving force for the crack to propagate toward the pore. It was shown that putting pore just in front of the tip of a crack approximately doubles the stress intensity factor at the crack tip. The volume fraction of porosity required to give crack deflection was found to be approximately 0.4. Despite of some simplified assumptions (replacement of sharp crack tip/microcrack interaction by pores), the model predictions is shown to be in agreement with experimental observations. The obtained analytical result means that in situation where no residual stresses are present, crack deflection mechanism can operate in high porous interlayer material.

The strength and crack propagation behavior of multilayered ceramics are not generally limited by only bulk properties but also by the local interaction of cracks with interfaces [10]. This can be controlled by such factors as the relative strength of the interface, the matrix and reinforcement phases as well as thermal expansion mismatch across the interface, which generally dictates the nature of the crack path. The selection of a crack trajectory in such layered structures is determined by a mutual competition between the direction of maximum mechanical driving force and the weakest microstructural path [11].

As described by Folsom et al. [12,13], the failure of laminated ceramic composites in flexural loading is characterized by a stepped stress–strain response. The sequential failure of each layer is accompanied by a load drop and a subsequent reloading to initiate failure in the next layer. Each load drop is associated with the failure of a single layer and the deflection of the crack along the interface. After each load drop, the stress in the next, unfractured layer is generally less, than its failure strength. Thus the remaining portion of the specimen must be reloaded to reinitiate the fracture process.

Clegg et al. [2] and Clegg [14] reported a similar dependence in the load–displacement curve of a notched-beam three point flexure test for laminated silicon carbide–graphite composites. In their studies, during load increases from the sawtooth minima to the maxima, a crack deflected along weak graphite interfaces (delamination), without growing across the lamina. It was observed that the failure of the next lamina normally occured well-behind the tip of the delamination crack and the crack had grown from some defect within the lamina. In addition, the tensile stress in the lamina calculated from the load was identical with the strength of an unnotched bar with the remaining thickness.

The influence of residual stresses distribution on character of crack propagation in laminar composites, containing layers of Y-TZP and either Al_2O_3 or a mixture of Al_2O_3 and $Y\text{-}ZrO_2$ was studied by Tomaszewski et al. [15]. Controlled crack growth experiments with notched beams of composites were done and showed the significant effect of barrier layer thickness and compositions on crack propagation paths during fracture. Distinct crack deflection in alumina layers was observed. As it occured the value of crack deflection angle was proportional to the layer thickness. In the case of layer thicknesses below 10 μm the crack was found to be undeflected. In barrier layers made of an oxide mixture crack deflection did not occur independently on layer thickness. These observations were explained by measurements of residual stress distribution in barrier layers. The magnitude of compressive stress in alumina layer on the layer boundary was independent on layer thickness. However the layer thickness affected the stresses gradient. The compressive stress was found to decrease from the boundary of layer to the center of alumina layer and here reached the minimum. The correlation between the value of crack deflection angle and the magnitude of stress gradient was found. In layer with thickness less than 10 μm crack did not deflect, and the compressive stress gradient reached very small value. In the barrier layers made of an oxide mixture, higher compressive stresses were found. However the distribution and local character of these stresses resulted in the lack of crack deflection independently on layer thickness.

Recently Kovar et al. have studied mechanical properties of Si_3N_4/BN multilayered ceramics, where the properties of the interface were adjusted by varying the composition of the BN interphase between the Si_3N_4 layers [16]. The strength and energy absorption of multilayered ceramics were measured, and the crack path was characterized as a function of the composition of the interphase. It was found that in materials with very high interfacial fracture resistance values (>80 J/m^2) no crack deflection was observed and very little energy was absorbed. Specimens with moderate interfacial fracture resistance values (50–80 J/m^2) exhibited crack deflection; however, the delamination cracks are short because the delamination cracks kink out of the interphase. These specimens also did not absorb much energy. Extensive delamination cracking and high energy absorption were observed only in materials that have the lowest interfacial fracture resistance (30–50 J/m^2).

The observations of Kovar et al. means that the energy-absorption capability of a material is not determined merely by whether or not crack deflection occurs. Rather, the extent of energy absorption is primarily influenced by the crack path after the initial crack deflection occurs. Specifically, the energy-absorption capability is greatly reduced when the delamination crack kink out of the interphase after travelling only a short distance. Long delamination distances are favored when the interfacial fracture resistance is low, the flaw size in the layers is small and the fracture resistance of the layers is high.

To date the approach has only been demonstrated for a limited range of systems due to the difficulties to find suitable interfacial materials. To be useful, such an interfacial material must be chemically compatible with the laminae, so that it can be used at an elevated temperature. Additionally, even when fracture toughness increases compared with that of monolitic ceramics, bending strength decreases because of weak continuous interlayer. Furthermore, because of the high anisotropy of layered materials with weak interfaces, the strength normal to the layers is low, and the area of application is strongly restricted.

Accounting for difficult technological problems to obtain laminates with interfaces (or interlayer) of controlled strength, the development of analytical methods to elucidate crack bifurcation phenomenon connected with compressive residual stress in the bulk of layer should lead to a better design for crack deflecting systems. Such bifurcation effect looks now as very promising in view of a recently discovered threshold strength of laminates [1], i.e., a strength below which there is theoretically zero probability of failure. Large flaws within the layers were observed to propagate and then arrest as they entered the compressive layers. For laminates of identical architecture, failure occured at the same applied tensile stress despite large differences in the initial size of the crack. This stress was the threshold strength, i.e., stress below which failure could not occur. Fracture surface observations indicated that the crack did not propagate straight across the compressive layers. Instead, the crack was observed to bifurcate within the compressive layer. It was confirmed [17] that crack bifurcation is observed for conditions of higher compressive stress and/or thicker compressive layers.

6. Conclusions

A method to analyze the crack deflection behavior of two component symmetric layered composites has been developed. It could be shown that a layer thickness regime exists where crack bifurcation occurs. This regime depends on the elastic constants of the layers, the number of layers, the thermal expansion factor difference and the temperature difference. No deflection of cracks occurs if the thickness of the first component layer is less than a characteristic thickness. For a given thickness of layers with tensile residual stresses there exists a thickness interval of the layers with compressive residual stresses where cracks bifurcate.

In the work the important case of specimens with fixed total thickness is considered. There are some features of crack bifurcation under these conditions. In this case the bifurcation area is situated between upper and low boundaries. There are cases when the low boundary of layer thickness to bifurcate is higher than the upper limitation due to fixed total thickness of sample. No bifurcation is in this case. More thickness of specimen determines more deflection area.

For layered sample with fixed thickness the useful parameter can be calculated. It is the critical number of layers. It is the maximum number of layers in specimen to occur the bifurcation under given conditions. If the sample has more number of layers it is no bifurcation. More thickness of specimen determines more number of laminar macrosructure variants to occur the bifurcation.

It should be noted the strong variations of l_c with temperature difference and with content of layers. It is important and efficient tool to control the bifurcation in laminates.

Acknowledgements

This work was supported by the European Commission. It is part of the Project "Silicon nitride based laminar and functionally gradient ceramics for engineering application", Proposal N 1CA2-1999-10109, Contract N 1CA2-CT-2000-10020, the program Copernicus-2.

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